THERMAL EFFECT OF AN ELECTRICAL CURRENT ON CONTACTS

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The temperature field within a contact is calculated analytically, with consideration of real current distribution within the contact body. The possibility of replacing the volume heat source by an equivalent surface heat source is demonstrated.

As is well known [1, 2], upon passage of an electrical current through a contact between conductors, where the effective conductor section decreases to a small conductive zone, there occurs intense heat liberation, accompanied by erosion of the contact surfaces under certain conditions.

The present study is a mathematical analysis of the temperature distribution in the current spread zone, which permits determination of this value as a function of current strength (density) and the thermophysical properties of the electrode material. The calculations are based on the distribution of current density flowing in a given area of finite radius into the electrode surface zone. This allows mathematical simulation of a thermal field most closely approximating the real case.

The calculations connected with consideration of the spatial distribution of current within the electrode body can be significantly simplified by taking a qualitatively new approach toward the problem, replacing the volume heat source by an equivalent surface source. One possible approach to this problem is described below, and curves which allow evaluation of the accuracy of the approximation (which proves to be quite high) are presented in Fig. 1.

1. Cylindrical Heat Source

It is assumed that the discharge is a cylindrical plasma column in contact with a semi-infinite electrode, unbounded in radius, forming a semispace. We will consider the stationary thermal regime described by the inhomogeneous Laplace equation,

$$\Delta T = -\frac{1}{\gamma} w(\vec{r}), \tag{1}$$

where Δ is the Laplace operator; w is the power liberated by current per unit electrode volume; γ is the thermal conductivity.

The solution of Eq. (1) in the most general case has the form [3]

$$T(\vec{r}) = \frac{1}{4\pi\gamma} \int \frac{w(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} d\vec{r}\,'. \tag{2}$$

In a cylindrical coordinate system with the z axis coinciding with discharge axis (so that the plane $z=0$ coincides with the electrode surface) we have

$$|\vec{r} - \vec{r}\,'| = [r^2 + r'^2 - 2rr'\cos(\varphi - \varphi') + (z - z')^2]^{\frac{1}{2}}, \tag{3}$$

where φ is the azimuth angle.

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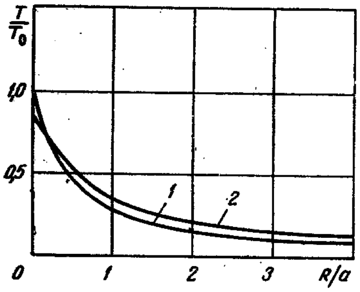

Fig. 1

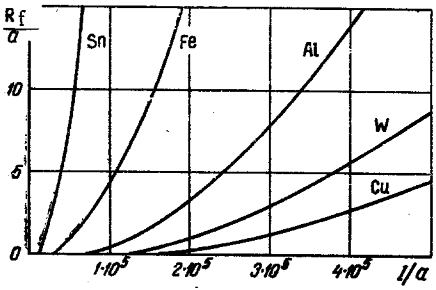

Fig. 2

Fig. 1. Comparative effect on electrode of volume (curve 1) and surface (curve 2) heat sources. Temperature measured in units $T_0 = I^2/a^2\sigma\gamma\pi^2$.

Fig. 2. Fusion-zone radius versus current for various materials.

We will assume that the heat source is cylindrically symmetrical $w(\vec{r}) = w(r, z)$ and perform integration over angle φ in Eq. (2). We write the denominator of the integrand in the form

$$|\vec{r} - \vec{r}'| = [r^2 + r'^2 + (z - z')^2]^{\frac{1}{2}} \left[1 - \frac{2rr' \cos(\varphi - \varphi')}{r^2 + r'^2 + (z - z')^2}\right]^{\frac{1}{2}} \tag{4}$$

and evaluate the second term in the final bracket. It is easy to see that the inequality

$$1 - \frac{2rr'}{r^2 + r'^2} = \frac{(r - r')^2}{r^2 + r'^2} > 0 \tag{5}$$

holds, and, consequently, this quantity cannot exceed unity. Thus the expression in brackets may be represented in the form of a series,

$$(1 - k\cos\theta)^{-\frac{1}{2}} = 1 + \frac{k}{2}\cos\theta + \frac{3}{8}k^2\cos^2\theta + \frac{15}{48}k^3\cos^3\theta + \frac{35}{128}k^4\cos^4\theta + \ldots, \tag{6}$$

where $\theta = \varphi - \varphi'$, $k = 2rr'/(r^2 + r'^2)$.

Substituting Eq. (6) in Eq. (2) and integrating over the angle θ', we find

$$\frac{1}{2\pi}\int_{\varphi}^{2\pi+\varphi} \frac{d\theta}{(1 - k\cos\theta)^{\frac{1}{2}}} \approx \frac{1}{[r^2 + r'^2 + (z - z')^2]^{\frac{1}{2}}}(1 + 0.125k^2 + 0.102k^4). \tag{7}$$

Since $k < 1$, the second and third terms in Eq. (7) are practically always small in comparison to unity, and they may be neglected, writing Eq. (2) in the form

$$T(r, z) = \frac{1}{2\gamma}\int_0^{\infty} r'dr' \int_{-\infty}^{\infty} \frac{w(r', z')\,dz'}{[r^2 + r'^2 + (z - z')^2]^{\frac{1}{2}}}. \tag{8}$$

Since we are interested in the temperature distribution in the semispace $z > 0$, we may transform Eq. (8), changing the limits of integration over z:

$$\int_{-\infty}^{\infty} \frac{w(z')\,dz'}{[r^2 + r'^2 + (z - z')^2]^{\frac{1}{2}}} = \int_0^{\infty} \left\{\frac{1}{[r^2 + r'^2 + (z - z')^2]^{\frac{1}{2}}} + \frac{1}{[r^2 + r'^2 + (z + z')^2]^{\frac{1}{2}}}\right\} w(z')\,dz'. \tag{9}$$

Expanding the right side of Eq. (9) in a power series

$$\frac{2zz'}{R^2 + R'^2} < 1,\quad R^2 = r^2 + z^2,\quad R'^2 = r'^2 + z'^2$$

and retaining only the first term of the expansion, we find

$$T(r, z) = \frac{1}{\gamma}\int_0^{\infty} r'dr' \int_0^{\infty} \frac{w(r', z')\,dz'}{(R^2 + R'^2)^{\frac{1}{2}}}. \tag{10}$$

We note that the error connected with the transformation from Eq. (9) to Eq. (10) does not exceed 3/8 in comparison to unity.

Equation (10) will be employed below for analysis of the thermal field in the electrode, created by volume w_V and surface w_S heat sources.

2. Volume Heat Source

We assume that the electrical current I flows into the semispace through a conductive area of radius a, located on the electrode surface in the plane z = 0. The distribution of current force lines within the electrode was found in [1] and has the form

$$J_r = \frac{I}{\pi a}\int_0^\infty J_1(\lambda a)\, J_1(\lambda r)\exp(-\lambda z)\, d\lambda,$$
$$J_z = \frac{I}{\pi a}\int_0^\infty J_1(\lambda a)\, J_0(\lambda r)\exp(-\lambda z)\, d\lambda, \tag{11}$$

where J_0 and J_1 are Bessel functions.

Correspondingly, the power liberated by the current in a unit volume is equal to

$$w_v = \frac{j^2}{\sigma}, \tag{12}$$

where σ is the electrical conductivity of the electrode material. Substituting w_V from Eq. (12) in Eq. (10), and considering Eq. (11), we obtain the following expression for temperature T(r, z):

$$T(r,\ z) = \frac{I^2}{\pi^2 a^2 \sigma\gamma}\int_0^\infty J_1(\lambda a)\, d\lambda \int_0^\infty J_1(\mu a)\, d\mu \int_0^\infty r' dr' \int_0^\infty dz' \exp[-(\lambda+\mu)z']\,[J_0(\lambda r')\, J_0(\mu r') + J_1(\lambda r')\, J_1(\mu r')]\, \frac{1}{(R^2+R'^2)^{\frac{1}{2}}}. \tag{13}$$

We will consider integrals over the variables r' and z', transforming to spherical variables $r' = \rho\cos\theta$ and $z' = \rho\sin\theta$. Then

$$\int \ldots d\vec{r}' = \int_0^\infty \frac{\rho^2 d\rho}{(R^2+\rho^2)^{\frac{1}{2}}} \int_0^{\frac{\pi}{2}} \exp[-(\lambda+\mu)\rho\sin\theta]$$
$$\times [J_0(\lambda\rho\cos\theta)\, J_0(\mu\rho\cos\theta) + J_1(\lambda\rho\cos\theta)\, J_1(\mu\rho\cos\theta)]\cos\theta d\theta. \tag{14}$$

The value of the integral is determined mainly by the exponential term, with the maximum contribution produced by the region $\theta \ll 1$. In light of this, the Bessel functions may be placed ahead of the integration sign, using the approximation $\theta \approx 0$. Then, using the relationship

$$\int_0^{\frac{\pi}{2}} \exp[-(\lambda+\mu)\rho\sin\theta]\cos\theta d\theta = \frac{1}{\rho(\mu+\lambda)}\{1-\exp[-(\lambda+\mu)\rho]\}, \tag{15}$$

we write Eq. (14) in the form

$$q(\lambda,\ \mu) = \frac{1}{\lambda+\mu}\int_0^\infty [J_0(\lambda\rho)\, J_0(\mu\rho) + J_1(\lambda\rho)\, J_1(\mu\rho)]\ \{1-\exp[-(\mu+\lambda)\rho]\}\, \frac{\rho d\rho}{(R^2+\rho^2)^{\frac{1}{2}}}. \tag{16}$$

In order to separate the dimensionless parameter, in Eq. (16) we perform the substitution $\rho = R\xi$. Then the integral takes on the form

$$q(\lambda,\ \mu) = \frac{R}{\lambda+\mu}\int_0^\infty [J_0(\lambda R\xi)\, J_0(\mu R\xi) + J_1(\lambda R\xi)\, J_1(\mu R\xi)]\ \{1-\exp[-(\lambda+\mu)R\xi]\}\, \frac{\xi d\xi}{(1+\xi^2)^{\frac{1}{2}}}. \tag{17}$$

We will now show that the integral of Eq. (17) has properties close to those of the delta function [4]. In fact, for $\lambda R \ll 1$ and $\mu R \ll 1$, the major contribution is produced by the region $\xi \gg 1$, and so we may take

$$\{1-\exp[-(\mu+\lambda)R\xi]\} \approx (\mu+\lambda)R\xi, \quad (1+\xi^2)^{-\frac{1}{2}} \approx \xi^{-1}$$

and, consequently, we have

$$q(\lambda, \mu) = R^2 \int\limits_0^\infty [J_0(\lambda R\xi) J_0(\mu R\xi) + J_1(\lambda R\xi) J_1(\mu R\xi)]\, \xi d\xi = 2\frac{1}{\lambda}\delta(\lambda-\mu). \tag{18}$$

In the region of high parameter values $\mu R \gg 1$ and $\lambda R \gg 1$ the maximum contribution to the integral is produced by the region $\xi \ll 1$, and therefore

$$q(\lambda, \mu) \approx \frac{1}{\lambda^2 R}\delta(\lambda-\mu). \tag{19}$$

Considering Eqs. (18), (19), we write the integral of Eq. (17) in the following form:

$$q(\lambda, \mu) = \frac{1-\exp(-2\lambda R)}{\lambda R}\frac{1}{\lambda}\delta(\lambda-\mu). \tag{20}$$

After substitution of Eq. (2) in Eq. (13) and integration over the variable μ with the aid of the δ-function [5] we find

$$T(R) = \frac{I^2}{\pi^2 a^2 R\sigma\gamma}\int\limits_0^\infty \frac{J_1^2(\lambda a)}{\lambda^2}[1-\exp(-2\lambda R)]\, d\lambda. \tag{21}$$

Unfortunately, the integral of Eq. (21) cannot be expressed in terms of elementary functions. Therefore we will consider its limiting cases. In the region $R \ll a$, setting $R = 0$ in Eq. (21) and expanding the exponential in a series, we obtain

$$T(R) = \frac{2I^2}{\pi^2 a^2\sigma\gamma}\int\limits_0^\infty J_1^2(\lambda a)\frac{d\lambda}{\lambda} = \frac{I^2}{\pi^2 a^2\sigma\gamma}. \tag{22}$$

At large distances $R \gg a$ we obtain [5]

$$T(R) = \frac{I^2}{\pi^2 a^2 R}\int\limits_0^\infty J_1^2(\lambda a)\frac{d\lambda}{\lambda^2} = \frac{4I^2}{3\pi^3 aR\sigma\gamma}. \tag{23}$$

In general form the integral of Eq. (21) can be reduced to full elliptic integrals [6],

$$T(R) = \frac{I^2}{\pi^2 a^2\sigma\gamma}\left\{\frac{4}{3\pi}\cdot\frac{a}{R} + 1 - \frac{4}{3\pi}\cdot\frac{R^2}{a^2}\left(1+\frac{a^2}{R^2}\right)^{\frac{1}{2}}\left[K(k) + \left(\frac{a^2}{R^2}-1\right)E(k)\right]\right\}, \tag{24}$$

where $k = a(a^2+R^2)^{-1/2}$. The functions $E(k)$ and $K(k)$ are tabulated in [6] and may be used for numerical calculations. A curve constructed on the basis of Eq. (24) is presented in Fig. 1.

Equation (24) permits determination of the dependence of the fusion zone radius R_f on current I. In fact, by setting $T = T_f$ in Eq. (24), we obtain R_f as a function of I. Results of numerical calculation for various metals are presented in Fig. 2.

It should be noted that in [2] the process of contact heating by an electrical current was examined, with consideration of the dependence of conductor resistivity on temperature, by simultaneous solution of the thermal-conductivity equation with a volume heat source and the equation for potential u, assuming that in the contact zone the potential remains constant.

Here we consider an analogous problem for the case where it is possible to neglect the dependence of conductor resistivity on temperature. But in contrast to [2] the thermal-conductivity equation with volume heat source is solved with consideration of the current density distribution in the contact, considering the potential at the boundary to be dependent on the coordinate r

$$u|_{z=0} = \frac{I}{\pi a\sigma}\int\limits_0^\infty \frac{J_1(\lambda a) J_0(\lambda r)}{\lambda}\, d\lambda.$$

Comparison of numerical results shows agreement far from ($r \gg a$) and divergence near ($r \ll a$) the contact area, as compared to the results obtained in [2]. This may be explained by the different character of the boundary conditions taken in determining the potential u.

3. "Surface" Heat Source

As was noted above, electrode heating is caused to a significant degree by the energy Qt, liberated in the interelectrode gap. In this case we can assume that the thermal field in the electrode is created by a "surface" heat source, located in the plane z = 0. In order to determine the power of this source, we will use the expression for boundary resistance to the current obtained in [1]:

$$R = \frac{8}{3\pi^2 \sigma a}. \tag{25}$$

Correspondingly, the total power liberated at the electrode boundary is equal to

$$Q' = R' I^2, \text{ where } R' = R + R_{\text{discharge}}. \tag{26}$$

Considering that the area of the influx zone is equal to πa^2 and the major voltage drop occurs in the narrow layer near the electrode boundary, we may represent the power density (without discharge I^2R) in the form

$$w_s = \begin{cases} \dfrac{8I^2}{3\pi^3 a^3 \sigma}\,\delta(z), & r \leqslant a, \\ 0, & r > a, \end{cases} \tag{27}$$

where $\delta(z)$ is the delta function.

Then to calculate the temperature generated by the "surface" source, we may use the general expression of Eq. (10). Substituting w_s from Eq. (27) in Eq. (10) and considering that integration over the variable z may be performed elementarily, we obtain

$$T(R) = \frac{8I^2}{3\pi^3 a^3 \sigma \gamma} \int_0^a \frac{r' dr'}{(R^2 + r'^2)^{\frac{1}{2}}}. \tag{28}$$

After integration over r' we finally obtain

$$T(R) = \frac{8I^2}{3\pi^3 a^2 \sigma \gamma} \left[\left(1 + \frac{R^2}{a^2}\right)^{\frac{1}{2}} - \frac{R}{a} \right]. \tag{29}$$

The temperature versus distance curve is presented in Fig. 1, from which one can compare the effect on the electrode of "surface" and volume heat sources, created by the current of an electric arc.

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